Derivatives of Composite Functions

$$\frac{d}{dx} x^n = n x^{n-1}$$

$$\frac{d}{dx} \sin x = \cos x$$

$$\boxed{\frac{d}{dx} \cos x = -\sin x}$$

$$\frac{d}{dx} \tan x = \sec^2 x$$

$$\boxed{\frac{d}{dx} \cot x = -\operatorname{cosec}^2 x}$$

$$\frac{d}{dx} \sec x = \sec x \tan x$$

$$\boxed{\frac{d}{dx} \operatorname{cosec} x = -\operatorname{cosec} x \cot x}$$

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Exercise 5.5

i) If $f(x) = \sqrt{4 - x^2}$, $\epsilon(-2,2)$ find $f'(x)$.

$$\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$$

$$f'(x) = \frac{d}{dx}\sqrt{4-x^2}$$

$$= \frac{1}{2\sqrt{4-x^2}} \frac{d}{dx}(4-x^2)$$

$$= \frac{1}{2\sqrt{4-x^2}} \times 0 - 2x$$

$$f'(x) = \frac{-2x}{2\sqrt{4-x^2}} = \frac{-x}{\sqrt{4-x^2}}$$

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2.(i) $\sqrt{2x-1}$

but

$y = \sqrt{2x-1}$

$\frac{dy}{dx} = \frac{d}{dx}\sqrt{2x-1}$

$= \frac{1}{2\sqrt{2x-1}} \frac{d}{dx}(2x-1)$

$= \frac{1}{2\sqrt{2x-1}} \times 2$

$= \frac{1}{\sqrt{2x-1}}$ Ans

ii) $(3x^2-9x+5)^9$

$y = (3x^2-9x+5)^9$

$\frac{dy}{dx} = \frac{d}{dx}(3x^2-9x+5)^9$

$= 9(3x^2-9x+5)^{9-1} \frac{d}{dx}(3x^2-9x+5)$

$= 9(3x^2-9x+5)^8 [6x-9]$ Ans

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3. (i) $\sqrt{3x+2} + \frac{1}{\sqrt{2x^2+4}}$

$$y = \sqrt{3x+2} + \frac{1}{\sqrt{2x^2+4}}$$

$$\frac{dy}{dx} = \frac{d}{dx}\sqrt{3x+2} + \frac{d}{dx}\frac{1}{\sqrt{2x^2+4}}$$

$$= \frac{1}{2\sqrt{3x+2}} \times \frac{d}{dx}(3x+2) + \frac{d}{dx}(2x^2+4)^{-1/2}$$

$$= \frac{1}{2\sqrt{3x+2}} \times 3 + \frac{1}{2}(2x^2+4)^{-\frac{1}{2}-1} \frac{d}{dx}(2x^2+4)$$

$$= \frac{3}{2\sqrt{3x+2}} + \frac{1}{2}(2x^2+4)^{-3/2} \times 4x$$

$$= \frac{3}{2\sqrt{3x+2}} + \frac{1}{(2x^2+4)^{3/2}} \times 2x$$

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ii) $sin^3x + cos^6x$

$$y = \sin^3 x + \cos^6 x$$

$$\frac{dy}{dx} = \frac{d}{dx}\sin^3 x + \frac{d}{dx}\cos^6 x$$

$$= 3\sin^2 x \frac{d}{dx}\sin x + 6\cos^5 x \frac{d}{dx}\cos x$$

$$= 3\sin^2 x \cos x + 6\cos^5 x(-\sin x)$$

$$= 3\sin^2 x \cos x - 6\sin x \cos^5 x \quad \text{Ans}$$

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4 (i) $\sin(x^2)$

$y = \sin(x^2)$

$$\frac{dy}{dx} = \frac{d}{dx} \sin(x^2)$$

$$= \cos x^2 \frac{d}{dx} x^2$$

$$\boxed{\frac{dy}{dx} = 2x \cos x^2}$$

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ii) $\cos\sqrt{x}$

$$y = \cos\sqrt{x}$$

$$\frac{dy}{dx} = \frac{d}{dx}\cos\sqrt{x}$$

$$= -\sin\sqrt{x}\,\frac{d}{dx}\sqrt{x}$$

$$= -\sin\sqrt{x} \times \frac{1}{2\sqrt{x}}$$

$$\boxed{\frac{dy}{dx} = -\frac{\sin\sqrt{x}}{2\sqrt{x}} \text{ Ans}}$$

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5.(i) $\sin(ax+b)$

$$y = \sin(ax+b)$$

$$\frac{dy}{dx} = \frac{d}{dx}\sin(ax+b)$$

$$= \cos(ax+b)\frac{d}{dx}(ax+b)$$

$$\boxed{\frac{dy}{dx} = a\cos(ax+b)}$$

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Derivatives of Inverse Trigonometric functions

$$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\cos^{-1}x = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\cot^{-1}x = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}\sec^{-1}x = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}\operatorname{Cosec}^{-1}x = \frac{-1}{x\sqrt{x^2-1}}$$

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v) $cot^{-1}\sqrt{x}$

$y = \cot^{-1}\sqrt{x}$

$$\frac{dy}{dx} = \frac{d}{dx}\cot^{-1}\sqrt{x}$$

$$= \frac{-1}{1+(\sqrt{x})^2}\frac{d}{dx}\sqrt{x}$$

$$= \frac{-1}{1+x} \times \frac{1}{2\sqrt{x}}$$

$$= \frac{-1}{(1+x)2\sqrt{x}} \quad \text{Ans}$$

vi) $(tan^{-1}x)^2$

$y = (\tan^{-1}x)^2$

$$\frac{dy}{dx} = \frac{d}{dx}(\tan^{-1}x)^2$$

$$\frac{dy}{dx} = 2(\tan^{-1}x)\frac{d}{dx}\tan^{-1}x$$

$$\frac{dy}{dx} = 2\tan^{-1}x \times \frac{1}{1+x^2}$$

$$\boxed{\frac{dy}{dx} = \frac{2\tan^{-1}x}{1+x^2}}$$

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3. Find the derivatives of the following functions:

i) $sec^{-1}(cosecx)$ ii) $tan^{-1}(Cotx)$

$$y = \text{Sec}^{-1}(\text{Cosec}\,x)$$

$$\frac{dy}{dx} = \frac{d}{dx}\,\text{Sec}^{-1}(\text{Cosec}\,x)$$

$$= \frac{-1}{\text{Cosec}\,x\sqrt{\text{Cosec}^2x-1}} \times \text{Cosec}\,x\,\text{Cot}\,x$$

$$\boxed{\frac{dy}{dx} = \frac{-\text{Cosec}\,x\,\text{Cot}\,x}{\text{Cosec}\,x\sqrt{\text{Cosec}^2x-1}}}$$ Ans

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4. If f(x)=$cot^{-1}(tanx)$, show that $f'(x)$=-1

$$f(x) = \cot^{-1}(\tan x)$$

$$f'(x) = \frac{d}{dx}\cot^{-1}(\tan x)$$

$$= \frac{-1}{1+\tan^2 x} \times \sec^2 x$$

$$= \frac{-1}{\sec^2 x} \times \sec^2 x$$

$$f'(x) = -1 \quad \text{Ans}$$

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5. If $y=sin^{-1}x+cos^{-1}x$, find $\frac{dy}{dx}$

$$y = \sin^{-1}x + \cos^{-1}x$$

$$\frac{dy}{dx} = \frac{d}{dx}\sin^{-1}x + \frac{d}{dx}\cos^{-1}x$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} = 0$$

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6. If f(x)=$sin^{-1}x + sec^{-1}\frac{1}{x}$, find $f'(x)$.

$$\frac{d}{dx}\frac{1}{x} = \frac{d}{dx}x^{-1}$$

$$= -x^{-1-1}$$

$$= -x^{-2}$$

$$f'(x) = \frac{d}{dx}\sin^{-1}x + \frac{d}{dx}\sec^{-1}\frac{1}{x}$$

$$= \frac{1}{\sqrt{1-x^2}} + \frac{1}{\frac{1}{x}\sqrt{\frac{1}{x^2}-1}}\,x^{-\frac{1}{x^2}}$$

$$= \frac{1}{\sqrt{1-x^2}} - \frac{1}{x\sqrt{\frac{1}{x^2}-1}}$$ Ans

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8. If $y = \sin^{-1}(\cos x) + 3\operatorname{cosec}^{-1}(\sec x)$ find $\frac{dy}{dx}$

$$y = \sin^{-1}(\cos x) + 3\operatorname{cosec}^{-1}(\sec x)$$

$$\frac{dy}{dx} = \frac{d}{dx}\sin^{-1}(\cos x) + 3\frac{d}{dx}\operatorname{cosec}^{-1}(\sec x)$$

$$= \frac{-1}{\sqrt{1-\cos^2 x}} \times \sin x - 3 \times \frac{1}{\sec x\sqrt{\sec^2 x - 1}} \times \sec x \tan x$$

$$= \frac{-\sin x}{\sin x} - \frac{3\tan x}{\tan x} = -1 - 3 = -4$$ Ans

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$$\sin 2x = 2\sin x \cos x$$

$$\cos 2x = 2\cos^2 x - 1$$

$$= 1 - 2\sin^2 x$$

$$= \cos^2 x - \sin^2 x$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

$$\sin 3x = 3\sin x - 4\sin^3 x$$

$$\cos 3x = 4\cos^3 x - 3\cos x$$

H.W

Learn Formula.

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